Performance Assessment Task
Houses in a Row Grade 3
This task challenges a student to use knowledge geometric and numerical patterns to identify and continue a pattern. A student must be able to use knowledge of patterns to evaluate and test a conjecture about how a pattern grows. A student must be able to model a problem situation with objects and use representations such as tables and number sentences to draw conclusions. A student must be able to explain and quantify the growth of a numerical pattern.
Common Core State Standards Math - Content Standards
Operations and Algebraic Thinking **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal parts.
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on

the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
3	2006	8	5	60%

Houses in a Row

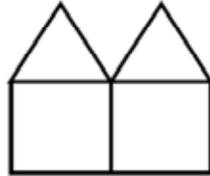This problem gives you the chance to:
• find a pattern in a sequence of diagrams
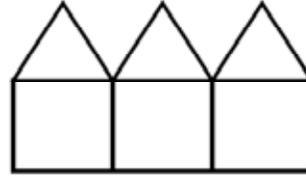• use the pattern to make a prediction

Lindsay uses toothpicks to make houses in a row.

1 house
6 toothpicks

2 houses
11 toothpicks

3 houses
16 toothpicks

Six toothpicks make one house, eleven toothpicks make two houses, and sixteen toothpicks make three houses.

1. Draw a diagram to show four houses in a row.

2. Lindsay makes a table to show the number of toothpicks needed to make different numbers of houses in a row.

Number of houses	1	2	3	4	5	6
Number of toothpicks	6	11	16			

How many toothpicks are needed to make four houses in a row?
Write your answer in Lindsay's table.

3. How many toothpicks are needed to make six houses in a row? _____

Explain how you figured it out.

_____

_____

_____

4. Lindsay has 41 toothpicks.

   How many houses in a row can she make? _____

   Explain how you figured it out.

   _____

   _____

   _____

   _____

5. Lindsay says, "*I need 55 toothpicks to make 11 houses in a row.*"

   Lindsay is wrong. Explain why she is wrong.

   _____

   _____

   _____

   _____

   How many toothpicks does Lindsay need to make 11 houses in a row?

   _____

8

Houses in a Row		Rubric

The core elements of performance required by this task are: • find a pattern in a sequence of diagrams • use the pattern to make a prediction Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Draws a correct diagram showing 4 houses in a row.	1	1
2. Gives correct answer: **21**	1	1
3. Gives correct answer: **31** Draws a correct diagram or gives a correct explanation such as: The numbers go up by 5s, so I added 10 to 21.	1 1	2
4. Gives correct answer: **8** Gives a correct explanation such as: Counts on from 6 houses need 31 toothpicks, 7 houses need 36 toothpicks, 8 houses need 41 toothpicks. Draws a correct diagram or gives a correct explanation such as: The first house needs 6 toothpicks. Each extra house needs 5 toothpicks. $41 - 6 = 35$, $35 \div 5 = 7$, $1 + 7 = 8$	1 1	2
5. Gives a correct explanation such as: The first house needs 6 toothpicks. Each extra house needs 5 toothpicks. $6 + 5 \times 10 = 56$ **or** The number of toothpicks shows a repeating pattern of 1 and 6 on the units digits. Gives correct answer: **56**	1 **or** 1 1	2
	Total Points	**8**

3rd Grade – Task 2: Houses in a Row

Work the task and examine the rubric.
What do you think are the key mathematics the task is trying to assess?

_____
_____
_____
_____

Look at your student papers.
Did your students have trouble with the visual discrimination and seeing the attributes of the houses?

- Did they draw discrete houses?
- Did they forget to include the vertical line separating the roof from the house?
- Did they add in extra details, like flowers or chimneys? What might these students not understand about the purpose of the task, diagram, or the mathematics? If you had time for an interview, what question would you want to ask the student?

How many of your students thought the pattern increased by 6's? What kinds of questions might push them to see the overlap as houses become connected?

Look at work for part 3, how many of your students put:

31	21	32 or 33	36	Over 36	Other

Now think about the strategies used by students who were successful, did they:

Draw & Count	Add 5's	Notice a pattern 1,6,1,6 . . .	Notice overlaps +12- 2	Other

The thinking for extending the pattern in part 4 can become more difficult. If students noticed that the pattern was going up by 5's, did they understand the meaning of the extra one? Do you think they could explain where the one came from by using the diagram?

Look at work for part 4. How many of your students put:

8	6	7	9	Other

What do you think they didn't understand about the pattern that led to the three common errors? What kinds of discussion could you facilitate with your students to get them to confront their misconceptions?

Do you think your students could describe how the pattern grows in words? Give a rule for finding the toothpicks for any number of houses? How might describing the pattern growth help them to find a rule? How would different ways of seeing the pattern result in different rules? Would all the rules give the same answer?

Understanding the Mathematics of Proportions and Functions with Constants
Consider the cost of buying t-shirts at Nancy's, $6 each, or J-mart, $1 to enter the shirt sale and $5 for each shirt.

Complete the tables.

Nancy's

Number of shirts	1	2	3	4	5	6
Total cost	6	12				

J-mart

Number of shirts	1	2	3	4	5	6
Total cost	6	11				

Now use your tables to answer these questions.
- Could you take the cost of 3 shirts and double it to find the cost of 6 shirts? Are your answers the same as the values in your chart? Why or why not?
- To find the cost of 15 shirts, could you take the cost of 6 shirts plus 6 shirts plus three shirts? Why or why not?
- To find the cost of 15 shirts, could you take the cost of 5 shirts and multiply by 3? Why or why not?

What is different about the two functions? Can you try to define why the addition and multiplication work for one store but not the other?

How does this relate to the mathematics in Houses in a Row?

Looking at student work on Houses in a Row:

A big piece of the mathematics of this task is seeing what happens when the houses connect, how groups of houses are different from the same number of individual houses. Student A thinks about each house having 6 toothpicks, but realizes that when the houses connect there are overlaps. Toothpicks need to be subtracted for each overlap. With questioning the student could probably explain that there are always one less overlaps than the number of houses. Algebraically, this pattern could be expressed as
6x – (x - 1)= toothpicks.

Student A

1. Draw a diagram to show four houses in a row.

2. Lindsay makes a table to show the number of toothpicks needed to make different numbers of houses in a row.

Number of houses	1	2	3	4	5	6
Number of toothpicks	6	11	16	21	26	31

How many toothpicks are needed to make four houses in a row?
Write your answer in Lindsay's table.

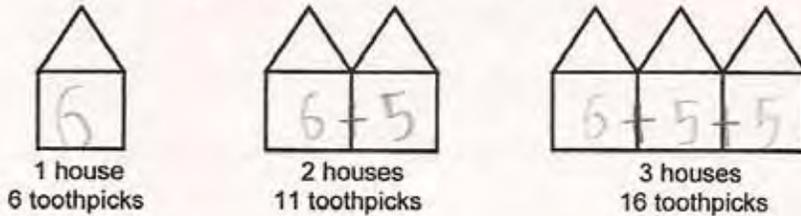3. How many toothpicks are needed to make six houses in a row? _31_ ✓
Explain how you figured it out.

21 + 12 = 33   33 - 2 = 31

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Another way of seeing the pattern is to think about the first house having six toothpicks and each additional toothpick house contains only 5 toothpicks because the left side is already made by the previous house. At a later grade level this might be expressed algebraically as toothpicks = 5(x-1) +6. See the work of Student B.
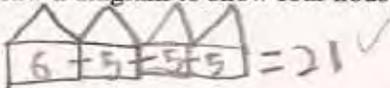
Student B

Lindsay uses toothpicks to make houses in a row.

| 1 house | 2 houses | 3 houses |
| 6 toothpicks | 11 toothpicks | 16 toothpicks |

Six toothpicks make one house, eleven toothpicks make two houses, and sixteen toothpicks make three houses.

1. Draw a diagram to show four houses in a row.

$6+5+5+5 = 21$ ✓

2. Lindsay makes a table to show the number of toothpicks needed to make different numbers of houses in a row.

Number of houses	1	2	3	4	5	6
Number of toothpicks	6	11	16	21	26	31 ✓

How many toothpicks are needed to make four houses in a row?
Write your answer in Lindsay's table.

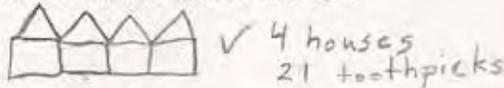3. How many toothpicks are needed to make six houses in a row? 31 ✓
Explain how you figured it out.

I did a short cut by starting at 21/4Houses and went 21+10=31 thats how I got the answer

As students move through the grades, they should start to think in groups or units other than ones. Student C is able to think about the pattern growing by a unit of 5 and then reason about how many "houses" or units are being added. This allows the student to move beyond the cumbersome strategies of drawing and counting or doing multiple additions.

Student C

1. Draw a diagram to show four houses in a row.

 ✓ 4 houses
21 toothpicks

2. Lindsay makes a table to show the number of toothpicks needed to make different numbers of houses in a row.

Number of houses	1	2	3	4	5	6
Number of toothpicks	6	11	16	21 ✓	26	31

How many toothpicks are needed to make four houses in a row?
Write your answer in Lindsay's table.

3. How many toothpicks are needed to make six houses in a row? __31 toothpicks__
Explain how you figured it out.

I knew that 4 house had 21 toothpicks, and
you need 5 more toothpicks for another house, and
I néded 2 more houses, so I did 5x2=10 then
10 + 21 = 31.

4. Lindsay has 41 toothpicks.

How many houses in a row can she make? __8__ ✓
Explain how you figured it out.

I knew 6 house had 31 toothpicks, and another
house has 5 toothpicks so I did 2×5=10 then
ten plus thirty-one = 41

5. Lindsay says, "I need 55 toothpicks to make 11 houses in a row."
Lindsay is wrong. Explain why she is wrong.

Lindsay is wrong because to make
11 houses you need 56 toothpicks. ✓
→ 41+15=56

How many toothpicks does Lindsay need to make 11 houses in a row?

56 ✓

The big mathematical idea is understanding that this function has a constant of 1, every house has 5 toothpicks + there is an additional 1 toothpick needed only by the first house (or there is a growth rate of 5 starting from 1 not zero). Student D understands this idea. Student E does not understand about the initial first toothpick. The student treats the pattern as the proportion 5x. *How would describing the pattern or using colors to show the pattern help Student E see his mistake?*

Student D

4. Lindsay has 41 toothpicks.

   How many houses in a row can she make?   8 houses ✓

   Explain how you figured it out.

   from 31, you keep adding 5 until you ✓
   get to the number 41.

5. Lindsay says, "I need 55 toothpicks to make 11 houses in a row."
   Lindsay is wrong. Explain why she is wrong.   ✓

   The 5×'s pattern starts with a
   six, so the 1's digit has to be
   an one or a six.

   How many toothpicks does Lindsay need to make 11 houses in a row?   ✓

                                      56 toothpicks

Student E

2. Lindsay makes a table to show the number of toothpicks needed to make different numbers of houses in a row.

Number of houses	1	2	3	4	5	6
Number of toothpicks	6	11	16	21	26	31

How many toothpicks are needed to make four houses in a row?
Write your answer in Lindsay's table.

3. How many toothpicks are needed to make six houses in a row? ___31___ ✓
Explain how you figured it out.

At first I 'dn't find a pattern, but then I realized that from 6 to 11, to 16 was just adding five every time

4. Lindsay has 41 toothpicks.

How many houses in a row can she make? ___8 houses___ ✓
Explain how you figured it out.

I figured it out by doing the 5x table until I got to 40. Then, I saw that it took me 8 (answer) times to get to 40!

If the table were a proportion (5x):						
1	2	3	4	5	6	
5	10	15	20	25	30	

Then it would be quite valid mathematically to take the second term (10) and multiply it by 3 to get 30 for the 6h term or add the 4th term + the 4th term + 3rd term to equal the 11th term (20+20+15 = 55 or 5 (11)= 55). However, this reasoning is not valid for functions with a constant. If numbers from a table with constants are added then the initial first toothpick is counted more than once. Students try to make generalizations such as these about patterns from work with proportional situations and don't realize that the procedures will not work for all patterns. See the work of Student F and G.

Student F

2. Lindsay makes a table to show the number of toothpicks needed to make different numbers of houses in a row.

Number of houses	1	2	3	4	5	6
Number of toothpicks	6	11	16	21	16	31

How many toothpicks are needed to make four houses in a row?
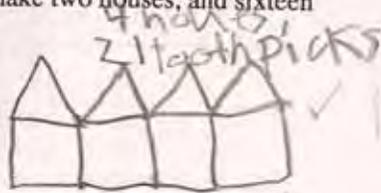Write your answer in Lindsay's table.

3. How many toothpicks are needed to make six houses in a row? __33__ X 0
Explain how you figured it out.

2 Houses is 11 toothpicks so 11 X3 =33   XO

Student G

Six toothpicks make one house, eleven toothpicks make two houses, and sixteen toothpicks make three houses.

1. Draw a diagram to show four houses in a row.

4 house, 21 toothpicks

2. Lindsay makes a table to show the number of toothpicks needed to make different numbers of houses in a row.

Number of houses	1	2	3	4	5	6
Number of toothpicks	6	11	16	21		

5. Lindsay says, "I need 55 toothpicks to make 11 houses in a row."
Lindsay is wrong. Explain why she is wrong.

I know she is wrong because
I made the 11 houses down
below first and it used
58 toothpicks.

How many toothpicks does Lindsay need to make 11 houses in a row?

21 + 21 + 16 = __58 toothpicks__

D
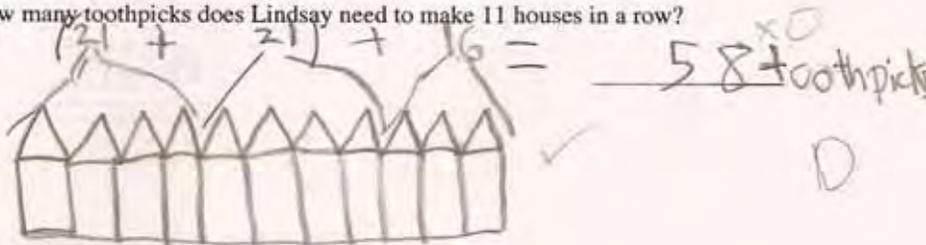
Some students have difficulty thinking about the growth, what stays the same and what changes. Student H sees that the first house has 6 toothpicks and incorrectly assumes that all the houses will have 6 toothpicks. While in part 2 the student can fill out a table the student doesn't connect the growth in the table to the growth in the pattern.

Student H

4. Lindsay has 41 toothpicks.

    How many houses in a row can she make?   _____6_____ x 0

    Explain how you figured it out.

    6 X 6 = 36 and 6 X 7 = 42 and she needs one more to make 7 houses. So she x 0 only could make 6 houses.

5. Lindsay says, "I need 55 toothpicks to make 11 houses in a row."

    Lindsay is wrong. Explain why she is wrong.

    Because she needs to have 11 more to make 11 houses. x 0

    How many toothpicks does Lindsay need to make 11 houses in a row?

    _____66_____ x 0

We want all students to have access to the problem using a variety of strategies. The focus of third grade number strand is to start seeing and thinking in groups, being able to see equal size groups as representing a unit that can be used in multiplication. In algebraic thinking students should be able to distinguish attributes of a geometric pattern and replicate the pattern. While many student need to draw and count, students should start to transition into more efficient strategies, like adding on or continuing a table. A few students will start to work towards generalization and finding and describing rules using multiplication and constants.

Third Grade

3rd Grade   **Task 2**   **Houses in a Row**

Student Task	Find a pattern in a sequence of diagrams. Use the pattern to make prediction. Use inverse operations to solve a problem. Develop a mathematical justification about why something is or isn't true.
Core Idea 3 Patterns, Functions, and Algebra	**Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships.** • Describe and extend geometric and numeric patterns. • Model problem situations with objects and use representations such as graphs and tables to draw conclusions. • Describe quantitative change. • Solve simple problems involving a functional relationship.

Mathematics in the Task:
- Ability to see and extend an existing geometric pattern and visualize the attributes.
- Ability to extend a table of numeric values based upon a specific geometric pattern.
- Ability to explain and quantify the growth of a numeric pattern.
- Ability to work backwards.
- Ability to reason and give mathematical justification for why a given answer is incorrect mathematically and then correct the error.

Based on teacher observation, this is what third graders knew and were able to do:
- Extend the pattern in pictures
- See the growth rate of 5
- Extend the pattern numerically in a table

Areas of difficulty for third graders:
- Understanding how the first term is different from the others
- Reasoning about the remainder of 1 when they divided in part 4
- Using drawing and counting or adding on accurately
- Communicating their strategy, how they figured out their answer

Strategies used by successful students:
- Drawing and counting
- Adding 5's or groups of 5
- Continuing the table
- Noticing that all odd houses end in 6, and all even houses end in 1 (while this was helpful in solving this specific problem, it would be difficult to use with large numbers)
- Multiplying by 6 and subtracting the overlaps
- Seeing the pattern 6 +5 +5 +5 …..

Task 2 - Houses in a Row

Mean: 4.96   StdDev: 2.81

Table 16: Frequency Distribution of MARS Test Task 2, Grade 3

Task 2 Scores	Student Count	% at or below	% at or above
0	879	7.4%	100.0%
1	1345	18.8%	92.6%
2	904	26.4%	81.2%
3	774	32.9%	73.6%
4	890	40.4%	67.1%
5	839	47.5%	59.6%
6	1374	59.1%	52.5%
7	1434	71.2%	40.9%
8	3412	100.0%	28.8%

Figure 25: Bar Graph of MARS Test Task 2 Raw Scores, Grade 3

MARS Task 2 Raw Scores
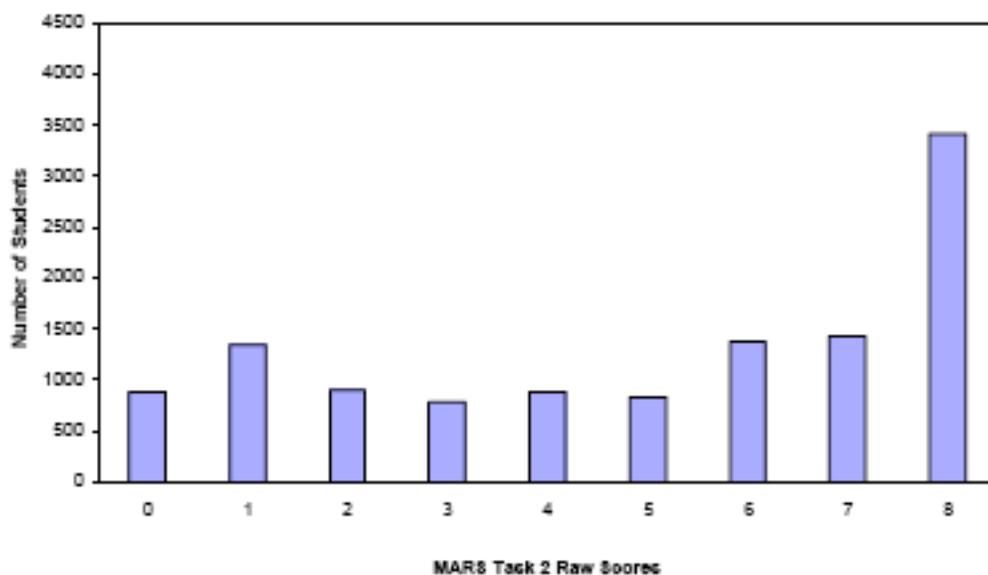
The maximum score available on this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, about 92%, could draw the fourth house. About 81% could also extend the values in the table. Many students, 61%, could draw the house, extend the table, find the value for the 6th house, and find the number of houses made with 41 toothpicks. More than half the students, 59%, could also either explain how they found the 6th house or how they found the number of houses for 41 toothpicks. Almost 29% could meet all the demands of the task including extending the pattern to 11 houses, and explaining mathematically how they found all their answers. 7% of the students scored no points on the task. All of the students in the sample with this score attempted the task.

Houses in a Row

Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task.	Students had difficulty with the geometric pattern. They often drew distinct, discrete houses rather than houses that were connected or they left out the toothpick separating the roof from the house.
1	Students could draw 4 toothpick houses.	Students had trouble thinking about what happens when houses are connected. They added by 6's instead of 5's.
2	Students could draw the 4 houses and find the total number of toothpicks.	They had difficulty extending the pattern to 6 houses. They may have made counting or computational errors. Many students could fill out the table correctly, but when asked about 6 houses in a row put the answer for 4 houses instead (9%).
4	Students could draw 4 houses, extend the pattern to 6 houses, and find the number of houses that could be made with 41 toothpicks.	Students had difficulty explaining how they found their answers for 6 houses or 41 toothpicks. In working backwards, many students could not explain where the remainder "one" came from when they divided by 5 or they got the correct answer by dividing by 6 with computation errors.
5	Students could draw the houses, extend the table, and then do one of the other three parts of the task. The split was pretty even between missing part 3, 4 or 5.	Students had difficulty explaining their thinking. They may have had trouble working backwards or extending the pattern to 11. Usually they tried to do one of the parts using groups of 6 or made mistakes in addition or subtraction.
7		Students had difficulty with the explanations in 3, 4, or 5.
8	Students could extend the pattern in words, tables, and by house number. Students could work backward from a number of toothpicks to find the number of houses. Students could express their thinking in words, calculations, or use pattern recognition. They were also able to make sense of the constant.	

Implications for Instruction

Students at this grade level need lots of activities to develop their visual discrimination. Activities with sorting help them to focus on attributes of different shapes. Having students pick their own categories for sorting adds the dimension of developing their own logic skills by forcing them to identify the visual attributes they are seeing. Students need opportunities to draw their own shapes and work with puzzles to develop a "feel" for how shapes are made and how angles fit together. Working with extending geometric patterns, sharpens the number of details that they pay attention to. Students should be asked to describe the pattern in words; telling what stays the same, what changes, and how the pieces fit together. This not only helps them with the visual discrimination; but also gives them important clues that leads to generalizations instead of recursive patterns.

In developing algebraic thinking in students of this age, teachers need to move students gradually away from drawing and counting strategies, which are inefficient and cumbersome, to thinking about growth rates and change. This process will lead students to developing adding on strategies or continuing tables. As students develop an understanding of the meaning of multiplication, they will hopefully start to think in groups (of five, in this case, but dependent on the specific problem). Students should start to make the connection that if they are adding 4 more units to the pattern, then they can multiply 4 times the growth rate and add that on.

While students at this grade level aren't expected to have a full understanding of a constant, they should notice where the pattern starts. "It is going up by 4's starting at 3 or going up by 5's starting at 6." The geometric context should help students reason about why the first term is different.

Ideas for Action Research:
Creating an Investigative Classroom, Learning to share for sense-making-

After giving students an opportunity to work this task alone, then they should compare their ideas with a group. Put up an example of a student dividing 41 by 6. Say you saw a student in another class solving the task this way. In pairs have students try to figure out what the student is thinking about. Then ask them if they think the student is right or wrong and why.

Now show them work like that of Student C, but take away the extra explanations. So for part 3: "I did 5 x2=10, then 10+21=31." For part 4, "I did 2 x 5 = 10, then 10 plus 31 = 41." For part 5, "I did 41 + 15 = 56." Ask them to figure out in their pairs what the student is doing. Why is the student multiplying? How does the student know how many to multiply by? Where does the 2 come from? Where does the 5 come from? What does the 15 mean?

Are there other examples that you think would be useful to put before students?

How does this process reengage students with the mathematics of the task? How does this promote useful discourse in the classroom? Why did I suggest taking away the labels or extra explanation off the work for student C? How does this push the thinking done by students?

Understanding Growth

A different approach to working the pattern problem would be to have students describe how the pattern grows: what stays the same, what changes, what happens when two pieces are combined? How is the first figure different than the next one that is added? If students can describe that the pattern is going up by 5's, see if they can outline the "5-ness" with a colored marker. See if students can develop a verbal rule to find for any house number.

Some students will see the pattern as growing by 6 every time, but with overlaps. Ask them if there is a way to predict the number of overlaps given any number of houses. Again, see if students can develop a verbal rule to find the number of toothpicks needed for any house number.

While not all students are ready to make generalizations, these types of questions can develop their logical reasoning skills so that at some future time they can move away from drawing and counting. What other questions might help students think about the relationship between their number calculations and the geometric pattern? Why is this important?

Both of these activities give students opportunities to talk meaningfully about mathematics and practice using mathematical vocabulary. It also emphasizes that mathematics is not just about getting an answer, but mathematics is also about sense-making and justification.